8th International Seminar on Optimization and Variational Analysis

Alicante, June 29–30, 2017

Universitat d'Alacant
Universidad de Alicante

UNIVERSITAS Miguel Hernández

GENERALITAT VALENCIANA
CONSELLERIA D'EDUCACIÓ, INVESTIGACIÓ, CULTURA I ESPORT

GOBIERNO DE ESPAÑA
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Venue

The conference will be held within the Campus of the University of Alicante, Building Aulario General I, Room **A1/0-03M** (ground floor).

The registration desk will be in the hall of Aulario General I, in front of the conference room.

The Poster Session will be held on Thursday, June 29, from 17:00 to 17:30 in the Room **A1/0-02M**.

Social Events

- Thursday 29th and Friday 30th at 13:15

  Lunch at *El monte del olivo restaurant*. Alicante University campus.

- Thursday 29th at 20:30

  Dinner at *La barra de César Anca*. Address: C/ Ojeda, 1. Alicante.

- Friday 30th at 20:30

  Dinner at *Restaurante Aldebarán*. Address: Real Club de Regatas, Muelle Poniente, 1. Alicante.

Schedule

Thursday, June 29

08:45 – 09:30	*Registration and Opening*
09:30 – 10:00	Jean-Baptiste Hiriart-Urruty (Toulouse Mathematics Institute) Mathematical *tapas o raciones*
10:00 – 10:30	Michel Volle (University of Avignon) Sensitivity and duality for multimap constrained optimization problems
10:30 – 11:00	Annamaria Barbagallo (University of Naples Federico II) Variational inequalities involving tensors
11:00 – 11:30	*Coffee Break*
11:30 – 12:00	Emilio Carrizosa (University of Sevilla) Fitting distance matrices to distances of (moving) convex bodies by DC optimization
12:00 – 12:30	Margarita Rodríguez (University of Alicante) On finite linear systems containing strict inequalities
12:30 – 13:00	Maria Josefa Cánovas (Miguel Hernández University of Elche) Sharp calmness constants in linear programming
13:15 – 15:15	*Lunch*
15:30 – 16:00	Roberto Lucchetti (Polytechnic University of Milan) Approximating quasiconvex functions with strictly quasiconvex ones in Banach space
16:00 – 16:30	Florent Nacry (University of Limoges) Solvability of discontinuous first and second order nonconvex Moreau's sweeping processes
16:30 – 17:00	Aris Daniilidis (CMM, University of Chile) A Morse-Sard result for Lipschitz selections
17:00 – 17:30	*Coffee Break* and *Poster Session*
17:30 – 18:00	Rubén Campoy (University of Alicante) A new projection method for finding the closest point in the intersection of convex sets
18:00 – 18:30	Carmen Galé (University of Zaragoza) A rank pricing problem
18:30 – 19:00	Cornel Pintea (Babeş-Bolyai University Cluj) Closed convex sets of Minkowski type
20:30	*Official Dinner*

Schedule

Friday, June 30

09:30 – 10:00	Juan Enrique Martínez-Legaz (Autonomous University of Barcelona) Minimization of quadratic functions on convex sets without asymptotes
10:00 – 10:30	Mikhail Solodov (IMPA) A globally convergent Linear-Programming-Newton method for piecewise smooth constrained equations
10:30 – 11:00	José Vicente-Pérez (University of Alicante) Radii of robust efficiency in robust multi-objective convex optimization
11:00 – 11:30	*Coffee Break*
11:30 – 12:00	Michel Théra (University of Limoges) Thirty minutes on directional Hölder metric regularity and its application to parametrized optimization
12:00 – 12:30	Lidia Huerga (Carlos III University of Madrid) Characterization of proper efficient solutions in non-convex multiobjective optimization with a polyhedral ordering cone
12:30 – 13:00	Maria Dolores Fajardo (University of Alicante) A comparison of alternative c-conjugate dual problems in infinite convex optimization
13:15 – 15:15	*Lunch*
15:30 – 16:00	Samir Adly (University of Limoges) On the proximity operator of the sum of two closed and convex functions
16:00 – 16:30	Maria Jesús Gisbert (Miguel Hernández University of Elche) Calmness of the optimal value function in linear programming
16:30 – 17:00	Pedro Pérez-Aros (University of Chile) A complete characterization of the subdifferential of convex integral functions
17:00 – 17:30	*Coffee Break*
17:30 – 18:00	Lina Mallozzi (University of Naples Federico II) Optimal transport methods in practical bilevel problems
18:00 – 18:30	Claudia Sagastizábal (IMPA) $\mathcal{VU}$ decomposition and partial smoothness for sublinear functions
20:30	*Closing Dinner*

Abstracts

On the Proximity Operator of the Sum of Two Closed and Convex Functions

Samir Adly

University of Limoges (France)

The main result of this talk is to provide an explicit decomposition of the proximity operator of the sum of two closed and convex functions. For this purpose, we introduce a new operator, called f-*proximity operator*, generalizing the classical notion. After providing some properties and characterizations, we discuss the relations between the f-*proximity operator* and the classical Douglas-Rachford operator. In particular we provide a one-loop algorithm allowing to compute numerically this new operator, and thus the proximity operator of the sum of two closed and convex functions. Finally we illustrate the usefulness of our main result in the context of sensitivity analysis of linear variational inequalities of second kind in a Hilbert space.

Joint work with: Loïc Bourdin (University of Limoges) and Fabien Caubet (University of Toulouse).

Variational Inequalities Involving Tensors

Annamaria Barbagallo

Department of Mathematics and Applications 'R. Caccioppoli', University of Naples Federico II (Italy)

The aim of the talk is to introduce a new class of variational inequalities on the tensor space. In particular, existence and uniqueness results are shown. Moreover, a class of variational inequalities in structured tensors is investigated. An important special case of this class is the nonlinear complementarity problem, recently introduced in the tensor-based form. Both the variational inequality problem and the nonlinear complementarity problem have application in finding the Nash equilibrium point of the n person noncooperative game and in the oligopolistic market equilibrium problem.

Joint work with: Serena Guarino Lo Bianco (University of Naples Federico II).

A New Projection Method for Finding the Closest Point in the Intersection of Convex Sets

RUBÉN CAMPOY

Department of Mathematics, University of Alicante (Spain)

In this talk we present a new iterative projection method for finding the closest point in the intersection of convex sets to any arbitrary point in a Hilbert space. This method, termed AAMR for averaged alternating modified reflections, can be viewed as an adequate modification of the Douglas–Rachford method that yields a solution to the best approximation problem. Under a constraint qualification at the point of interest, we show weak convergence of the method. In fact, the so-called strong CHIP fully characterizes the convergence of the AAMR method for every point in the space. The scheme is shown to be strongly convergent for affine constraints. We report some promising numerical experiments where we compare the performance of AAMR against other projection methods for finding the closest point in the intersection of pairs of finite dimensional subspaces.

Joint work with: F.J. Aragón Artacho (University of Alicante).

Sharp Calmness Constants in Linear Programming

MARIA JOSEFA CÁNOVAS

Center of Operations Research, Miguel Hernández University of Elche (Spain)

This talk is focussed on some recent advances in the study of the calmness property of ordinary (finite) linear programs under canonical perturbations (i.e., perturbations of the objective function coefficient vector and the right-hand side of the constraint system). We show that the expression for the calmness modulus of the argmin mapping given in [1] is indeed a calmness constant in a certain neighborhood, which is also provided, of the nominal minimizer. We emphasize the fact that the expressions for both the (sharp) calmness constant and the neighborhood can be easily computed as far as they only depend on the nominal data (in this sense we call them point-based). As an intermediate step, we prove that an analogous fact occurs for the feasible set mapping associated with linear inequality systems under right-hand side perturbations: the point-based expression for the corresponding calmness modulus given in [2] turns out to be a calmness constant in a certain neighborhood, for which we also give a point-based expression. We also show that this result cannot be extended to general convex systems.

References

[1] M.J. Cánovas, R. Henrion, M.A. López, J. Parra: Outer limit of subdifferentials and calmness moduli in linear and nonlinear programming. *J. Optim. Theory Appl.* 169 (2016) 925–952.

[2] M.J. Cánovas, M.A. López, J. Parra, F.J. Toledo: Calmness of the feasible set mapping for linear inequality systems. *Set-Valued Var. Anal.* 22 (2014) 375–389.

Joint work with: J. Parra and F.J. Toledo (Miguel Hernández University of Elche) and J .-J. Rückmann (University of Bergen, Norway).

Fitting Distance Matrices to Distances of (moving) Convex Bodies by DC Optimization

Emilio Carrizosa

Institute of Mathematics, University of Seville (Spain)

A finite set J, and a set of distance matrices $\{\Delta_t : t = 1, \ldots, T\}$ in J are given, as well as a compact convex $S \subset \mathbb{R}^n$ set containing 0 in its interior. We pose the problem of finding, for each $j \in J$ and $t \in \{1, \ldots, T\}$, a set of the form $c_{jt} + \lambda_{jt} S$ such that the distance between sets fit as much as possible Δ_t, and some smoothness along t is obtained.
This problem, which extends the standard Multidimensional Scaling Analysis in Data Analysis, is written as a global optimization problem whose objective is a dc function. A suitable dc decomposition allows us to use the DCA algorithm in a very efficient way. Examples will be presented.

Joint work with: Vanesa Guerrero (University of Seville) and Dolores Romero Morales (Copenhagen Business School, Denmark).

A Morse-Sard Result for Lipschitz Selections

Aris Daniilidis

CMM, University of Chile (Chile)

We establish the following result: the set of Clarke critical values of a finite (or infinite countable) selection over a family of C^k-continuously differentiable functions has measure zero. The result applies in particular to max-type functions and yields a straightforward application to semi-infinite programming.

Joint work with: Luc Barbet and Marc Dambrine (Université de Pau et des Pays de l'Adour).

A Comparison of Alternative c-Conjugate Dual Problems in Infinite Convex Optimization

MARIA DOLORES FAJARDO

Department of Mathematics, University of Alicante (Spain)

In this work, we obtain a Fenchel–Lagrange dual problem for an infinite dimensional optimization primal one, via perturbational approach and using a conjugation scheme called c-conjugation instead of classical Fenchel conjugation. This scheme is based on the generalized convex conjugation theory. We analyse some inequalities between the optimal values of Fenchel, Lagrange and Fenchel–Lagrange dual problems and we establish sufficient conditions under which they are equal. Examples where such inequalities are strictly fulfilled are provided. Finally, we study the relations between the optimal solutions and the solvability of the three mentioned dual problems.

Joint work with: José Vidal (Technische Universität Chemnitz, Germany).

A Rank Pricing Problem

CARMEN GALÉ

Department of Statistical Methods, University of Zaragoza (Spain)

Management and economic planning analyze how to sell the right product to the right customer for the right price. Companies use pricing strategies to maximize their profits. The knowledge and the understanding of customer behavior allow the companies to assess the price responsiveness of the different customer segments. This work focuses on a price optimization problem which aims to setting the prices of a set of products taking into account the customer preferences. This problem involves a hierarchical structure which is modeled as a bilevel program. The leader or upper decision maker establishes the prices of the products bearing in mind the reaction of the customers, that is, the consumer purchasing behavior. Each customer or lower decision maker, once the prices are known, selects the product which maximizes his/her preferences according to his/her budget constraint. From the bilevel formulation of the problem, several single level formulations are developed and strengthening valid inequalities are proposed. A computational study is carried out to analyze the efficiency of the proposed formulations.

Joint work with: Herminia I. Calvete (Universidad de Zaragoza) and Martine Labbé (Université Libre de Bruxelles).

Calmness of the Optimal Value Function in Linear Programming

Maria Jesús Gisbert

Center of Operations Research, Miguel Hernández University of Elche (Spain)

The final goal of the paper presented in this talk consists in computing/estimating the calmness moduli from below and above of the optimal value function restricted to the set of solvable linear problems. Roughly speaking these moduli provide measures of the maximum rates of decrease and increase of the optimal value under perturbations of the data (provided that solvability is preserved). This research is developed in the framework of (finite) linear optimization problems under canonical perturbations; i.e., under simultaneous perturbations of the right-hand-side (RHS) of the constraints and the coefficients of the objective function. As a first step, part of the work is developed in the context of RHS pertubations only, where a specific formulation for the optimal value function is provided. This formulation constitutes the starting point in providing exact formulae/estimations for the corresponding calmness moduli from below and above. We point out the fact that all expressions for the aimed calmness moduli are conceptually tractable (implementable) as far as they are given exclusively in terms of the nominal data.

Joint work with: M.J. Cánovas, J. Parra and F.J. Toledo (Miguel Hernández University of Elche).

Mathematical *tapas o raciones*

Jean-Baptiste Hiriart-Urruty

Paul Sabatier University, Toulouse (France)

Mathematical tapas... but what are tapas? *Tapas* is a Spanish word (in the Basque country, on would also say *pintxos*) to call small savory dishes typically served in bars, with some drinking, shared with friends in a relaxed ambiance. The offer is varied: it may be meat, fish, vegetables, ... Each of the guests of the party selects the tapas he likes most at the moment. This is the spirit of the mathematical tapas that we offer here.

The tapas that we present here are in the same spirit as in our previous volume, published with the same title, but which targeted the undergraduate level (see reference at the end). However, the tapas in this volume are more substantial, they therefore could be called *raciones* instead. They consist of mathematical questions to answer, exercises (more than long problems, in their spirit), of various types. They concern mathematics ranging from the undergraduate to the graduate level (roughly speaking, this corresponds to the end of the third year and the fourth year at university[1]); they do not cover the whole spectrum of mathematics, of course. Clearly, they reflect the mathematical interests and teaching experience of the author:

Metric, normed, Banach, inner-product and Hilbert spaces: basic calculus with distances and norms, convergent or Cauchy sequences, oddities in the infinite-dimensional setting;

Differential calculus: calculus rules, applications to unconstrained optimization problems in various settings;

[1]Called “Licence 3” and “Master 1” in the European Higher Education system.

Integration: examples of effective calculations of integrals, nothing on the theoretical aspects;

Matrices: especially symmetric and positive semidefinite ones, links with quadratic functions, interplays with geometry, convexity and optimization;

Convexity: convex sets, convex functions, their use in optimization;

Optimization or "variational" problems: arising in geometry (triangles or polyhedrons), unconstrained or constrained ones (with equality constraints mainly).

To reflect the variety of mathematics, there is no specific ordering on topics: tapas are more or less "randomly" presented, even if some gatherings have been carried out (for example on mean value theorems, on weakly converging sequences in Hilbert spaces, etc.)

How have they been chosen?

Firstly, because "we like them" and have tested them. In other words, each tapa *reveals something*: it could be an interesting inequality among integrals, an useful or surprising property of some mathematical objects (especially in the infinite-dimensional setting), or simply an elegant formula... We are just sensitive to the aesthetics of mathematics. Secondly, because they illustrate the following motto: "if you solve it, you learn something". During our career, we thought to hundreds of students and, therefore, posed thousands of exercises (in directed sessions of exercise solving, for exams, etc.); but we have not kept here (standard) questions whose objective is just to test the ability of calculating a gradient, sums of series or integrals, eigenvalues, etc. We therefore have limited our choice of tapas for this second volume to the (symbolic) number of **222**.

Where have they been chosen?

We observed that, year after year, some questions give rise to interest or surprise among students. Our mathematical tapas are chosen among them, as also in my favorite journals posing such challenges: the *American Mathematical Monthly*, the French mathematical journals entitled *Revue de la Filière Mathématique* (formerly *Revue de Mathématiques Spéciales*) and *Quadrature*. From time to time, I posed or solved questions posted in these journals. Some longer or more substantial tapas have been reconstituted from those already present, in a similar form, in the books that I have written in French in the past. However, for many tapas, I must confess that I have simply forgotten their origin or history.

How are they classified?

Like for guides for restaurants, each tapa has one, two or three stars.

- One star (★). Tapas of the first level, for students at the end of their undergraduate studies.

- Two stars (★★). Tapas of a more advanced level. That does not mean that solving them necessarily requires more expertise or wit than for one-starred ones, but sometimes just more maturity (or prerequisite definitions) in mathematics.

- Three stars (★★★). The upper level in the proposed tapas, typically for students in the first years of graduate studies. Some may be tough and need more chewing.

We admit that this classification is a bit arbitrary for some of the problems, as it depends on the reader's background.

How are they presented?

Each tapa begins with a *statement*, of course. The statement may contain the answers to the posed questions; this is the case when the questions or proposals are formulated like "Show that" or "Prove that".

There are no detailed solutions to all questions, as that would have inflated this booklet by a factor three or four. Moreover, in mathematics, there is no uniform and unique way to write down answers. But, to help solving the posed challenges, we have proposed *hints*... They suggest a path to follow. A question without any indication could be labelled "can't be done" or too time-consuming...; the same question with “spoon-fed” steps could be considered too easy. We have tried to strike a balance between the two postures which reflects the variety of the tapas. Of course, an interested reader is asked to try to chew and swallow the tapas without having recourse to the hints.

When they are not integrated in the statements, we provide *answers* to questions, numerical results for example. From time to time, we add some *comments*: on the context of the question, on its origin, on a possible extension.

Not all questions in mathematics have known answers... To illustrate that, we have added at the end 8 open problems or conjectures. This is our *open bar* section... the reader helps himself. Of course, there are neither hints nor answers for them since they are unknown... We have chosen these open problems at the level and on topics considered in this volume (numbers, real analysis, matrices, optimization, ...): easy to understand, concerning various areas of mathematics, original for some of them. They are marked with the symbol (♣♣♣).

In spite of our efforts, some misprints or even mistakes may have slipped in; we just hope that they are not irreparable.

An essential characteristic of mathematics is to be universal and thus international. So, imagine a student or someone who has some knowledge in mathematics (say, undegraduate level) in seclusion for some time on an isolated island, or just put into jail. With a book like the one containing these tapas, he might even enjoy his time and savour some of them.

Buen provecho!

References

J.-B. Hiriart-Urruty, *Mathematical tapas*, Volume 1 (for Undergraduates). Springer (October 2016).

J.-B. Hiriart-Urruty, *Mathematical tapas*, Volume 2 (From Undergraduate to Graduate level). Springer (Book to appear in October 2017).

Characterization of Proper Efficient Solutions in Non-convex Multiobjective Optimization with a Polyhedral Ordering Cone

Lidia Huerga

Department of Economics, Carlos III University of Madrid (Spain)

In the framework of a multiobjective optimization problem in which the ordering cone is assumed to be polyhedral, we characterize proper efficient solutions through nonlinear scalarization, and by using a type of polyhedral dilating cones. For this aim, no convexity assumptions are required. These dilating cones are constructed by perturbing the matrix that defines the ordering cone, so they are easy to manage and they let us obtain more attractive scalarization results from a computational point of view. Finally, in the particular case when the feasible set is given by a cone constraint, we derive necessary and sufficient conditions for proper efficient solutions via a scalar nonlinear Lagrangian.

Joint work with: César Gutiérrez (Universidad de Valladolid) and Vicente Novo (Universidad Nacional de Educación a Distancia).

Approximating Quasiconvex Functions with Strictly Quasiconvex ones in Banach space

ROBERTO LUCCHETTI

Polytechnic University of Milan (Italy)

In this paper we show how to approximate a quasiconvex function with a sequence of strictly quasiconvex functions in a reflexive Banach space X. An important role in our approximation procedure is played by a real valued convex function defined on X, and parameterized by a pair of closed bounded convex sets, which is a generalization of the classical Minkowski functional on X; for this reason, we investigate some of its properties. In particular, we prove the continuity of this map, seen as a function acting from a specific family of pairs of closed convex subsets of X, to the space of the real valued continuous functions on X. In the domain space we use the (bounded) Hausdorff topology, while the target space is endowed with the topology of the uniform convergence on bounded sets. Our results also need to approximate a closed convex set, in the sense of the bounded Hausdorff topology, with a sequence of strictly convex sets. The result particularizes to Hausdorff topology if the limit set is bounded.

Joint work with: Monica Milasi (Università di Messina, Italy).

Optimal Transport Methods in Practical Bilevel Problems

LINA MALLOZZI

Department of Mathematics and Applications 'R. Caccioppoli', University of Naples Federico II (Italy)

Optimal transport theory is widely used to solve problems in mathematics and different areas of the sciences. We formulate some problems in applied economics as two-stage schemes studied by using optimal transport tools. More precisely, two-stage optimization models corresponding to economic equilibrium problems are presented. A distribution of citizens in an urban area, where a given number of services must be located, is given. Citizens are partitioned in service regions such that each facility serves the costumer demand in one of the service regions.

At first, it is assumed that the demand is totally satisfied and in the spirit of a market survey, a social planner divides the market region into a set of service regions in order to minimize the total cost: the objective is to find the optimal location of the services in the urban area and the related costumers partition. Existence results are obtained by using optimal transport mass tools. Then, a bilevel formulation is considered related to an optimal monopoly pricing where customers have the option of not purchasing the good and the utility for purchasing the good at a given price may be random. In this case the problem is solved via partial transport mass theory.

The talk relies on [1,2].

References

[1] L. Mallozzi, A. Passarelli di Napoli: Optimal transport and a bilevel location–allocation problem. *J. Global Optimization* 67 (2017) 207–221.

[2] G. Carlier, L. Mallozzi: Optimal monopoly pricing with congestion and random utility via partial mass transport. *J. Math. Anal. Appl.* online 5 January 2017.

Minimization of Quadratic Functions on Convex Sets without Asymptotes

Juan Enrique Martínez-Legaz

Department d'Economia i d'Història Econòmica, Universitat Autònoma de Barcelona (Spain)

The classical Frank and Wolfe theorem states that a quadratic function which is bounded below on a convex polyhedron P attains its infimum on P. We investigate whether more general classes of convex sets F can be identified which have this Frank-and-Wolfe property. We show that the intrinsic characterizations of Frank-and-Wolfe sets hinge on asymptotic properties of these sets.

Joint work with: Dominikus Noll (Université de Toulouse, France) and Wilfredo Sosa (Universidade Católica de Brasília, Brazil).

Solvability of Discontinuous First and Second Order Nonconvex Moreau's Sweeping Processes

Florent Nacry

University of Limoges (France)

In 1971, J.J. Moreau introduced the following evolution problem (called "sweeping process") with $F \equiv 0$

$$\begin{cases} -\dot{u}(t) \in N(C(t); u(t)) + F(t, u(t)) & \lambda\text{-a.e. } t \in [0, T], \\ u(0) = u_0, \end{cases} \tag{1}$$

for a convex moving set $C(\cdot)$, a multimapping F and where $N(\cdot;\cdot)$ denotes the normal cone in the sense of convex analysis. Later, C. Castaing ([2]) developed existence results for the second order differential inclusions (with $F \equiv 0$ and $C(t, x) = C(x)$)

$$\begin{cases} -\ddot{u}(t) \in N(C(t, u(t)); \dot{u}(t)) + F(t, u(t), \dot{u}) & \lambda\text{-a.e. } t \in [0, T], \\ u(0) = b, \dot{u}(0) \in C(0, b). \end{cases} \tag{2}$$

It has been well recognized that such differential inclusions (1) and 2 play an important role in elastoplasticity and dry friction theory.

This talk is devoted to the study of existence result for the perturbed (i.e., $F \not\equiv 0$) first (resp., second) order sweeping process in the discontinuous framework that is, $u(\cdot)$ (resp., the velocity $\dot{u}(\cdot)$) will be only required to be of bounded variation. Here, the set $C(\cdot)$ (resp., $C(\cdot, \cdot)$) is assumed to be prox-regular ([6]) and move in a bounded variation way. The normal cone involved $N(\cdot, \cdot)$ is one of the usual normal cones of variational analysis (proximal, Fréchet, Mordukhovich limiting or Clarke).

References

[1] S. Adly, F. Nacry: An existence result for discontinuous second-order nonconvex state-dependent sweeping processes. *Submitted*.

[2] C. Castaing: Quelques problèmes d'évolution du second order. *Sém. Anal. Convexe*, Montpellier, 1988, Exposé No 5.

[3] J.F. Edmond, L. Thibault: BV solutions of nonconvex sweeping process differential inclusions with perturbation. *J. Differential Equations* (2006).

[4] J.J. Moreau: Rafle par un convexe variable I. *Travaux Sém. Anal. Convexe*, Montpellier, 1971.

[5] F. Nacry: Perturbed BV sweeping process involving prox-regular sets. To appear in *J. Nonlinear Convex Anal.*

[6] R.A. Poliquin, R.T. Rockafellar, L. Thibault: Local differentiability of distance functions. *Trans. Amer. Math. Soc.* (2000).

Joint work with: Samir Adly (University of Limoges).

A Complete Characterization of the Subdifferential of Convex Integral Functions

Pedro Pérez-Aros

University of Chile and University of O'Higgins, (Chile)

We study some subdifferentiation proprerties of integral functionals, given in the form

$$\hat{I}_f(x) := \int_T \max\{f(t, x(t)), 0\} d\mu(t) + \int_T \min\{f(t, x(t)), 0\} d\mu(t), \quad x(\cdot) \in \mathfrak{X},$$

with the associated integrand $f : T \times X \to \overline{\mathbb{R}}$ being measurable in (t, x) and convex in x, where $(T, \mathcal{A}, \mu)$ is a complete σ-finite measure space and $\mathfrak{X}$ is a linear space of $\mathcal{A}$-measurable functions with values on a locally convex space X.

In this work, we confine ourselves to the space of constant functions, in which case I_f becomes the continuous sum

$$x \in X \to I_f(x) = \int_T f(t, x) d\mu(t).$$

Then we give a characterization of ϵ-subdifferential of the integral functional I_f in terms of the ϵ-subdifferential of the data functions $f_t := f(t, \cdot)$. This provides a generalization of a well-known formula given by Ioffe-Levin [2].

Others formulae for the sum rule and for infinite series of convex functions will be presented. We shall also investigate exact rules to characterize the subdifferential of the integral functional I_f at a point $x \in X$ in terms of measurable selections $x^*(t) \in \partial f_t(x(t))$ for measurable functions $x(\cdot)$ close to the point x. This result is compared to the work of [1] and Lopez-Thibault [3].

References

[1] A.D. Ioffe: Three theorems on subdifferentiation of convex integral functionals. *J. Convex Anal.* 13 (2006) 759–772.

[2] A.D. Ioffe, V.L. Levin: Subdifferentials of convex functions. *Trudy Moskov. Mat. Obsc.* 26 (1972) 3–73.

[3] O. Lopez, L. Thibault: Sequential formula for subdifferential of integral sum of convex functions. *J. Nonlinear Convex Anal.* 9 (2008) 295–308.

Joint work with: Rafael Correa and Abderrahim Hantoute (CMM, University of Chile).

Closed Convex Sets of Minkowski Type

CORNEL PINTEA

Department of Mathematics, Babeş-Bolyai University Cluj (Romania)

We provide several characterizations of Minkowski sets, i.e. closed, possibly unbounded, convex sets which are representable as the convex hulls of their sets of extreme points. The equality between the relative boundary of a closed convex set containing no lines and its Pareto-like associated set ensures the Minkowski property of the set. In two dimensions this equality characterizes the Minkowski sets containing no lines.

Joint work with: J.E. Martínez-Legaz (Universitat Autònoma de Barcelona).

On Finite Linear Systems Containing Strict Inequalities

MARGARITA RODRÍGUEZ

Department of Mathematics, University of Alicante (Spain)

This talk deals with linear systems containing finitely many weak and/or strict inequalities whose solution sets are referred to as evenly convex polyhedral sets. The classical Motzkin Theorem states that every (closed convex) polyhedron is the Minkowski sum of a convex hull of finitely many points and a finitely generated cone. In this sense, similar representations for evenly convex polyhedra have been recently given by using the standard version for classical polyhedra. In this talk, we provide a new dual tool that completely characterizes finite linear systems containing strict inequalities and it constitutes the key for obtaining a generalization of Motzkin Theorem for evenly convex polyhedra.

Joint work with: J. Vicente-Pérez (University of Alicante).

$\mathcal{VU}$ Decomposition and Partial Smoothness for Sublinear Functions

Claudia Sagastizábal

National Institute for Pure and Applied Mathematics (Rio de Janeiro, Brazil)

$\mathcal{VU}$ decomposition and partial smoothness are related notions for generic nonsmooth functions. Sublinear functions are widely used in optimization algorithms as they themselves contain important structures. We discuss how to characterize several concepts related to partial smoothness for this type of functions. In particular, for a closed sublinear function h we introduce a relaxed decomposition depending on certain $\mathcal{V}_\varepsilon$ and $\mathcal{U}_\varepsilon$ subspaces that converge continuously to the $\mathcal{V}$ and $\mathcal{U}$ counterparts as ε tends to zero. To the new $\mathcal{V}_\varepsilon\mathcal{U}_\varepsilon$ decomposition corresponds a relaxed smooth manifold that contains the manifold where h is partly smooth.

Joint work with: Shuai Liu and Mikhail Solodov (IMPA, Rio de Janeiro, Brazil).

A Globally Convergent Linear-Programming-Newton Method for Piecewise Smooth Constrained Equations

Mikhail Solodov

National Institute for Pure and Applied Mathematics (Rio de Janeiro, Brazil)

The LP-Newton method for constrained equations, introduced some years ago, has powerful properties of local superlinear convergence, covering both possibly non-isolated solutions and possibly non-smooth equation mappings. We develop a related globally convergent algorithm, based on the LP-Newton subproblems and linesearch for the equation's infinity-norm residual. In the case of smooth equations, global convergence of this algorithm to B-stationary points of the residual over the constraint set is shown, which is a natural result: nothing better should generally be expected in variational settings. However, for the piecewise smooth case only a property weaker than B-stationarity can be guaranteed in general. We then develop an additional procedure for piecewise smooth equations that avoids undesirable accumulation points, thus achieving the intented property of B-stationarity.

Thirty minutes on Directional Hölder Metric Regularity and its Application to Parametrized Optimization

MICHEL THÉRA

University of Limoges (France)

In this talk, I will report some new results obtained with my vietnamese co-authors Nguyen Huu Tron and Huynh Van Ngai on regularity of multifunctions through various characterizations of directional Hölder/Lipschitz metric regularity. The results are based on the concepts of slope and coderivative. By using these characterizations, we show that directional Hölder/Lipschitz metric regularity is stable, when the multifunction under consideration is perturbed suitably. Applications of directional Hölder/Lipschitz metric regularity to investigate the stability and the sensitivity analysis of parameterized optimization problems will be also discussed.

Radii of Robust Efficiency in Robust Multi-objective Convex Optimization

JOSÉ VICENTE-PÉREZ

Department of Economics, University of Alicante (Spain)

This talk deals with multi-objective convex programming problems in the face of data uncertainty in either the objective function, or the constraints, or both. We consider highly robust weakly efficient solutions, that is, robust feasible solutions which are weakly efficient for any possible instance of the objective function within a specified uncertainty set, providing an exact formula for the radius of robust highly weak efficiency guaranteeing the existence of this type of solutions under linear perturbations of the objective functions. We also consider minmax robust weakly efficient solutions, that is, the weakly efficient solutions of the robust counterpart, providing optimality conditions and an existence theorem, and showing the existence of this type of solutions under sufficiently large linear perturbations of the objective functions. The mentioned results, which are new even for scalar optimization problems, are specified for the particular case of linear robust multi-objective programming.

Joint work with: M.A. Goberna (University of Alicante), V. Jeyakumar and G. Li (University of New South Wales, Australia).

Sensitivity and Duality for Multimap Constrained Optimization Problems

Michel Volle

University of Avignon (France)

We characterize the lower semicontinuity of the value function associated with an optimization problem whose constraint is defined by a multimap. Horizontal and vertical perturbations are considered simultaneously. A special attention is paid to the case when the objective function is linear. The settings of a multimap which is nondecreasing with respect to a solid convex cone (conic inequality constraints for instance), graph convex, or convex valued are of particular practical interest, and several results by Champion and Ban-Song are thus generalized.

Joint work with: Emil Ernst (Aix-Marseille University).

Poster Session

Lipschitz Modulus of Fully Perturbed Linear Programs

Francisco Javier Toledo

Center of Operations Research, Miguel Hernández University of Elche (Spain)

This talk deals with the Lipschitz-like (Aubin) property of the argmin mapping associated with linear programs under full perturbations (i.e., perturbations of all coefficients). First, we characterize the property itself and, in a second step, we provide a formula for computing the exact Lipschitz modulus. The direct antecedent of this work can be found in [1], which is developed in the framework of canonical perturbations; i.e., keeping fixed the left-hand-side of the constraints. We emphasize the fact that allowing perturbations of the left-hand-side entails notable differences with respect to referred work. In particular, the arguments of this paper are strongly based on the very recent results about calmness moduli of both the feasible set and the argmin mappings.

References

[1] M.J. Cánovas, F.J. Gómez Senent, J. Parra: On the Lipschitz modulus of the argmin mapping in linear semi-infinite optimization. *Set-Valued Anal.* 16 (2008) 511–538.

Joint work with: M.J. Cánovas and J. Parra (Miguel Hernández University of Elche).

Total Variation Image Reconstruction Problems on Smooth Surfaces

José Vidal

Technische Universität Chemnitz (Germany)

In this poster we present a work on a similar total variation image reconstruction approach [Rudin, Osher, Fatemi (Physica, 1992)] for images defined on smooth surfaces. The problem is defined in terms of quantities intrinsic to the surface, being therefore independent of the parametrization. We prove that the space of functions of bounded variation is embedded in the space of square integrable functions, so the integrals in the fidelity and regularization terms of the target primal problem will be well defined. We also provide a rigorous analytical framework for this model and its Fenchel predual in functional spaces. It is shown that the Fenchel predual of the total variation problem is a quadratic optimization problem with pointwise inequality constraints on the surface. We formulate a function space interior point approach to solve the predual problem. Finally, we analyze the well-posedness of the barrier approximations providing necessary and sufficient optimality conditions for its unique solution.

Joint work with: Roland Herzog (Technische Universität Chemnitz), Heiko Kröner (University of Hamburg), Stephan Schmidt and Marc Herrmann (Julius-Maximilians-Universität Würzburg).